640. D'Amore B. (2008). Leonardo and the lunula. Testo di presentazione della cartella realizzata da Giunti Editore come omaggio ai partecipanti al convegno internazionale: ICMI 1908/2008. The First Century of the International Commission on Mathematical Instruction. Reflecting and Shaping the World of Mathematics Education. 5-8 marzo 2008. Roma. Accademia dei Lincei e Istituto della Enciclopedia Italiana. Il testo è uscito anonimo.

On the $10^{\text {th }}$ November 1494, Luca Pacioli's Summa de Arithmetica, Geometria, Proportioni et Proportionalità is published; Leonardo buys a copy straight away that he orders from Milan and pays 119 soldi (as he annotates, with his usual precision, in the Atlantic Code, f. 104 r). He studies it and draws thousands of inspirations till he summarizes the chapters relative to the theory of proportions in the Madrid Code (8936). But he is fascinated mainly by geometry and in particular by the squaring of the circle and the theory of the lunulae.
The real meeting between Leonardo and Luca takes place in Milan in 1496, when the latter is entrusted by the Duke as a public teacher of mathematics. The interest for geometry, already so rooted in Leonardo, grows out of all proportions as Luca reveals it to him... His geometry becomes more learned, the problems proposed to himself as a challenge are almost always drawn from Pacioli's work, often in turn drawn from Euclid. Leonardo, in particular, falls in love of the golden section, the "divine proportion", and of "geometry's classical problems"; for example he faces the so called Delos problem, i.e. the construction of a cube that doubles the volume of a given one, even if, in the Atlantic Code, f. 161 r, Leonardo considers a cube whose side is 4 (therefore the volume is 64) and another one whose side is 5 (therefore the volume is 125) declaring that the latter doubles the former.
For sure, one of Leonardo's most long-living passions, to which he dedicated many energies and many folios of several codes, is the squaring of curvilinear figures; the beautiful f. 455 r of the Atlantic Code contains more or less 180 exercises on the construction of figures; he inscribes a square into a circle and obtains circular segments.
It looks as if, instead of using a mathematical skill, Leonardo is using his imagination typical of a painter, knowing he is overwhelmingly gifted with it. Above all there are semicircles, to save space.
This kind of drawings finally leads Leonardo to the study of lunulae that had been typical of Greek geometry and dominated by the figure of Hippocrates of Chios (V century b. C.), probably Pythagorean, one of the most renowned mathematicians of his century. He wrote a text entitled Elements of which we have nothing left but this title.


The lunula is the hatched part

Well, almost with only pictorial techniques, but also with the mathematics learned thanks to Luca, Leonardo finally arrives at considerable mathematical projects, transforming polygons into parts of a circle.
Enthusiast, as he often is in these cases, he even plans a treatise on curvilinear figures (in the Atlantic Code) that he entitles Libro d'equazione; such treatise should have continued and broadened previous studies entitled De equalità di superficie ineguali and Scienza de equiparanzia, already started theoretically in the Foster Code (f. 111 r and 112 r , respectively).
These activities on the lunulae and curvilinear surfaces sometimes give free rein to the pictorial imagination of Leonardo who indulges in real "geometrical games", as he himself declares. In these games there clearly is a more pictorial will than a mathematical one, even if underneath always lie hidden problems of transformation of rectilinear figures into curvilinear ones, and vice versa: the topic to which Leonardo gave his greatest contribution of all his mathematical production.

